CHAPTER 2

On Technical Progress and the Gains and Losses from Outsourcing

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INTRODUCTION

Today, comparative advantage in trade between two countries is no longer determined primarily by the happenstance of the distribution of natural resources between the trading partners. This change does not call for fundamental revision of Ricardo’s comparative advantage analysis, but it does bring out more emphatically some attributes and implications of the analysis that are often misunderstood (though Ricardo himself was sufficiently astute to avoid these misapprehensions). In particular, in this chapter we will show the following are particularly significant possibilities in the contemporary world and its trading activities.

(1) Enhanced productivity in the industries of a trading partner can be harmful to the home country. Although the effect of such development can be beneficial to both countries when the partner’s productive activities are initially primitive and inefficient, beyond some degree of success in the catch-up process it becomes damaging to the welfare of the home country.

(2) There is a dominant and dominated relationship possible between two trading countries, a relationship that is beneficial for the dominant economy and damaging to the dominated one.

(3) A country can attain a dominant position without intervention if the trading partner is in a properly undeveloped state by the happenstance of history or other fortuitous circumstances. But the development lag also can be imposed by
its trading partner’s mercantilist activities, if they succeed in destroying foreign industries.

(4) There is an inherent conflict of interest not only between a nation in a dominant trade position and its relatively advanced trading partner, but also between that dominant partner and the combined interests of a hypothesized two-country world.

(5) While a country cannot gain a dominant position just by enhancing the productivity of its industries, it can avoid a dominated position and assure a beneficial outcome for itself and the world, on balance, by developing a sufficient subset of its own industries and not allowing them to be destroyed.

**Misconceptions on Comparative Advantage**

Many of the misconceptions related to our subject stem from imputing to the powerful principle of comparative advantage alleged implications that simply do not flow from Ricardo’s analysis.

**Perfect specialization.** To a considerable degree, these misunderstandings are attributable to the failure to take into account the role of demand, relative and absolute, for the traded commodities at issue. Let us note some of the ways in which this omission affects the discussion.

First, we will observe that the “law” of comparative advantage does not imply that in a regime governed by the principle, there will be full specialization, with each country serving as the sole producer of its own set of exported commodities. Of course, simple observation confirms that reality displays nothing even approximating such an outcome. But the elementary textbooks persist in providing such artificial illustrations, using two producing parties and two commodities. The usual parable postulates that one of the two traders (Ricardo’s England) is less efficient in producing each of these two commodities, A and B, but is more inefficient in producing B. If one producer makes only good A, while the other supplier (Ricardo’s Portugal) makes only good B, both will gain by trading.

So far so good, but suppose that the demand for good B (e.g., wine) is minuscule and that for good A (e.g., cloth) is relatively abundant. The world demand for wine from Ricardo’s Portugal is limited by fear of inebriation, while the cold winters elicit an enormous cloth market. Then, surely, Portugal will end up producing both commodities, and specialization will no longer be the state of affairs. A moment’s thought surely will confirm that there is nothing surprising about this observation. This, in turn, leads to a possibility that is less obvious.

**Mutual gains.** Expository discussions of comparative advantage theory assert, or at least imply, that in a world market so constituted, the gains from trade of two countries
engaged in that activity must be mutual—that is, each participant must share in the benefits and, presumably, derive a share that is not negligible. But this too surely is affected by the relative strength of the demand for the two products being traded.

However, even this more modest conclusion does not hold everywhere. Indeed, it does not normally hold at all in Ricardo's cloth–wine parable. Remember that production in this two-good case is strictly linear. Suppose, then, that demand is such as to lead Portugal to produce both wine and cloth. Then, in equilibrium, the ratio of the prices must equal the ratio of their marginal costs to Portugal and, by linearity, also be equal to the ratio of their average costs.

A moment’s thought then readily confirms that whatever gains Portugal obtains from its trade with England, it would gain exactly equally by producing both items for itself in a state of autarky. Thus, Portugal gains nothing by engaging in trade!

**Mutual benefits from enhancement of productivity in one country.** That conclusion, in itself, admittedly entails a very special case and is of interest only in its warning that the standard conclusions in the comparative advantage arena may not be correct. But a related and more significant conclusion that productivity improvements in one country can actually harm its trading partner already has been noted by eminent analysts. For instance, this was pointed out by Professor J. R. Hicks in his inaugural address at Oxford University (Hicks 1953), by the present authors in Gomory and Baumol (2000)—which contains a significant part of the reasoning that is the basis for this chapter, and most recently in Samuelson (2004).

**Diagrammatic representation**

This analysis is easily formalized and summarized in a version that is derived directly from the standard mathematization of the Ricardian model. (A mathematical representation of that model is given in appendix 1 of this chapter.) From this, the equilibrium solution with any given vector of parameter values can be obtained. Each such solution yields for each of the trading countries, among other things, the national incomes of each country (in a simplified world composed of two countries that trade with one another), the price of each commodity, the quantity produced, and the equilibrium magnitude of exports and imports. From these numbers, one can easily and directly compute the national incomes of the two countries, \( Y_1 \) and \( Y_2 \), and from the utility functions included in the model we can calculate the total utility, \( U_i \) (\( i = 1, 2, w \)), thereby obtained by each country and the world. It also is convenient to formulate another variable, \( Z_i = Y_i/(Y_1 + Y_2) \), which gives the share of country \( i \) in world income.

The single equilibrium just described can be represented by three points on a simple graph (see Figure 2.1), with \( Z_1 \) on the horizontal axis indicating for any given equilibrium the division of incomes between the two countries. For that value of \( Z_1 \), the corresponding three points indicate respectively the utility levels, \( U_1, U_2, \) and \( U_w \), achieved with that share of world income by Country 1, Country 2, and the world in total.
By varying the productivity levels for the different products in the two countries, we obtain different equilibria, represented by different sets of the three points in Figure 2.1. Since the production of workers in any of the industries is bounded above by what can be produced using the best current technology, the quantities of the various goods that can be produced are all bounded from above. It follows immediately that the utilities of the two countries also must be bounded from above. So all possible equilibrium points will be bounded from above by three upper boundary curves—one for each country and one for the world. In Figure 2.1, these are labeled $U_w U_w$, $U_1 U_1$, and $U_2 U_2$, respectively.

It is plausible that the outcomes generally should look like the one in Figure 2.1. We expect the curves to be roughly hill-shaped, with the world curve ($U_w U_w$) having its peak between those of the two countries. While careful calculation via the model shows that the boundary curves for models with only a small number of traded commodities are rather jagged, intuition leads us to expect the curves to approximate those rounded shapes, and this is confirmed by calculation. The value of $Z_1$ on the horizontal axis gives Country 1’s share of world income. It is implausible that the world will be at its most prosperous when one country produces almost every product (as is true at $Z_1 = 0$ or $Z_1 = 1$), so that the world curve will attain its summit, $W^*$, more toward the middle of the graph.

Next, we note that $1^*$, the summit point of Country 1, will always lie to the right of the world summit, $W^*$, while the reverse will always be true for the summit of Country 2. That is, at $1^*$, Country 1 will be better off than it would be at the level of $Z_1$ that is optimal for the world, and Country 2 will have suffered a substantial loss of utility. An intuitive explanation is straightforward. As we move to the right from $W^*$, toward point $1^*$, we can think of the change as one in which the world output pie has shrunk (i.e., the world utility boundary has declined). But Country 1, by obtaining a larger slice of this smaller pie, still benefits until this trade-off goes too far. In that situation, Country 2 evidently obtains a smaller share of a smaller pie. Hence, self-interest will drive Country 1 to try to move to the right of $W^*$, at the expense of Country 2, and the reverse is evidently desired.
by Country 2. In short, each country wants to improve its payoff by acquiring a share of the world’s industries that is greater than that which is best for the world as a whole. (See appendix 3 for more discussion of the shape of the boundary curves.)

We can think of the country that succeeds in this tug-of-war as the country that has achieved dominance, and we can consider the other nation to be the dominated country. We use this labeling later in this chapter, where we note that each country hopes to be the one that is dominant, achieving this desired state through enhancement of its productivity levels in some of the commodities it seeks to export, rather than importing them—as it did in the past.

For two countries with labor forces of equal size, dominance is obtained by producing considerably more of the world’s value than the trading partner. This means that the dominant country is the producer in a set of industries, \( S_1 \), for which the total world demand is large, while the trading partner is the producer in a set of industries, \( S_2 \), for which the total world demand is small. This desirable state for the dominant country can be brought about if it has some insurmountable natural advantage in all the industries of \( S_1 \), or, as is more likely in the modern world, if it has acquired state-of-the-art abilities in the industries of \( S_1 \) that its trading partner does not have.

This state of affairs can be established easily if the trading partner has never developed these industries. It also can be created through the use of mercantilist practices, such as subsidies or currency manipulation, which destroy the trading partner’s \( S_1 \) industries.

**Outsourcing: how it can benefit the outsourcer’s firms and harm the home country’s inhabitants**

We have just seen how a trading country can benefit at the expense of the general welfare of society by acquiring industries from other countries. In this process, Country A gains not only at the expense of Country B, its trading partner, but also via a decline in welfare in the world economy, generally. Now let us look at this story in more detail, bringing outsourcing explicitly into the picture. In addition to simply losing its value-added in a particular industry through the loss of that industry to a trading partner, a country can lose value added in the industry through outsourcing, while retaining all or part of the production of the final product. To make the story more suggestive, let us deal with two countries that we will call “India” and the “United States.” We begin our story with an “India” of former years, when it had a very limited cadre of trained engineers and technicians among its population. Because their employment opportunities were very limited, their wages were very low. Suddenly there is a change in regime in that country, and the opportunities for education in the sciences and engineering expand markedly, offering some gain in income for those who graduate from these new programs. However, those wages are still very low, compared to those in the “United States” for people with similar capabilities.
At this point, managements in “American” firms realize that investment in “India” offers profitable prospects. Laboratory work, industrial design, and other such activities can be carried out there by trained and capable individuals whose wages, though significantly higher than before, are still significantly lower than those in the “United States.” The sequel is obvious. Wherever they can be done profitably, these activities in the “United States” will be threatened with outsourcing to “India.”

The benefits to the large “American” firms, which thereby have their outputs produced more cheaply, are obvious. They can sell their products at lower prices that attract larger sales volumes or they can choose to cut their prices only modestly and, thereby, yield greater profit margins than before.

The plausible implications for wages in the two countries are also fairly obvious. As usual, the pressure of competition upon two suppliers of a given product is also clear: The wage levels will be driven toward equality, reducing them in the “United States” and raising them in “India.” These wage changes then will extend to other industries in the two countries. For instance, increased demand for labor in the activities outsourced to “India” will put upward pressure on the wages of workers with comparable skills, working on other “Indian”-produced outputs for both domestic consumption and export.

Other consequences are not so simple. For example, if the outsourcing occurs during a relatively prosperous period in the “United States,” no decline in the level of employment will necessarily result there. Wages will be driven downward, but jobs will not dry up. Still, that is no blessing for the labor force. In a period of recession and unemployment, outsourcing can indeed exacerbate the shortage of jobs.

There is a second, less obvious consequence for the well-being of the “American” public. True, as often argued, consumers in the “United States” will benefit from the decreased prices of the outsourced commodities. But there are other products that the “United States” had been importing from “India” before the outsourcing began. With “Indian” wages rising generally, the prices of those other “Indian” manufactures also will move upward. Systematic analysis, such as that provided in Gomory and Baumol (1998), shows that as more and more activities are transferred from the “United States” to an initially impoverished “India,” both will benefit initially, but eventually—when the status of “India” has improved sufficiently—the net effect of the further transfer of industries from the “United States” to “India” will be a fall in the economic well-being in the former and a rise in that of the latter. This in fact is the message from the shape of the curves in Figure 2.1.

**TOWARD GENERALIZATION: DOMINANT AND DOMINATED ECONOMIES VERSUS THE INTERMEDIATE WORLD IDEAL**

We already have mentioned the concept of dominating and dominated economies, and it is not difficult to conclude that in the real world, at least until recently, the economy of
the actual India was dominated by that of the United States. Indeed, this characterization of their relative positions doubtless continues to be valid today. But it is equally clear that India and other emerging economies (e.g., China, Brazil, etc.) hope to move in the direction of dominance, though the fields in which they seek to advance differ from country to country.

In addition, we already have noted that, analytically speaking, we can think of the acquisition of industries by one country, at the expense of the country in which those industries formerly were located, as the archetypical instrument by which the formerly dominated country can strengthen its dominance over its trading partners. We also have seen (1) that there is an intermediate distribution of industries between two such countries at which the welfare of their combined populations is maximized, but that (2) each country will be motivated to seek to acquire more than this ubiquitously optimal share of industries because this gain in industries will increase the wealth of the gaining country, at the expense of the country that loses its industries. That is, the gains of the former will be exceeded by the losses imposed upon its trading partner.

In such a world in which economies are not driven by eleemosynary ambitions and charitable aims, each country can be expected to act in a way that is expected to bring gains to itself, at the expense of its trading partner. However, purely profit-driven outsourcing is a threat to a home country’s well-being that works in the opposite direction. That is, outsourcing from dominant Country A to dominated Country B facilitates realization of the latter’s ambition to transform itself from dominated toward dominator. As Country A does so, on the initiative of its firms with an international presence, those firms clearly will have the opportunity to profit, but at the expense of the welfare of the inhabitants of their home country.

**Innovation and enhanced education as remedies?**

In searching for the cure for America’s unbalanced trade, many observers place their faith in Silicon Valley and other centers of innovation. They note that past technological progress, with its origins in the United States, constituted a successful defense against imported products supplied by “cheap foreign labor.” Based on this, they conclude that the cure for today’s and tomorrow’s problems stemming from the current conditions of trade entails increasing the number of students who specialize in subjects useful for furthering innovation, such as engineering and physics. The sequel to this, it is believed, will be acceleration in the appearance of new products and other innovations in the United States, which, in turn, will provide more jobs and better wages.

There is evidently some substance to these contentions, but matters are not quite so simple. As we will argue next, these views fail to account adequately for important factors that influence trade, such as demand, the varying productivity performance of different economic sectors, and more recently the outsourcing of jobs. Most notably, this
viewpoint devotes inadequate attention to the role of domestic demand for labor in the relevant sectors of the economy.

**Does the United States need to educate more engineers and applied scientists?**

First, let us consider the education component of the argument that the training of more engineers and scientists will lead to more innovation and, thereby, spur economic growth. It is undoubtedly true that the spectacular innovative performance of the United States played an important role in its balance of payments in the period after the Second World War and even in the pre-Depression 1920s, when, as today, it also was widely noted that the US economy faced the rivalry of “cheap foreign labor.”

The United States seemingly had no shortage of engineers and scientists in these earlier periods, and we may do well to question the need for more now. Some evidence of the answer to this question is, in fact, readily available. The market, working via the supply–demand mechanism, surely does its bit to indicate which occupations are in need of more workers—an item in short supply will dependably be one whose price is relatively high. However, a comparison of the earnings of engineers and physicists with those of doctors, lawyers, and business executives surely indicates that the market for engineers and physicists does not seem to be driven by severe current shortage. True, 2009 data from the United States Bureau of Labor Statistics suggest that the average engineer actually earns a little bit more than the average business manager and that the average physicist makes a little bit more than the average lawyer. But if we compare the average of engineers’ and physicists’ average hourly wages with the average of doctors’, lawyers’, and business managers’ average hourly wages, we find that the latter are demonstrably higher than the former (US Bureau of Labor Statistics, 2009). In short, there appears to be a good market for the services of those who lead the innovation process, but surely no evidence of a desperate shortage of engineers and applied scientists.

**Will continued innovative performance ensure good jobs and high wages?**

The recent performance of the Chinese economy indicates that rapid growth can occur in the absence of spectacular invention. Indeed, that country, which arguably was the world’s leader of invention during Europe’s Middle Ages, until very recently apparently devoted little effort to inventive activity. Ironically, the general population of China benefited little from that earlier outpouring of domestic invention. Only today, with an economy driven by manufacturing, is its growth rate outpacing much of the world.

As China’s case hints, if an economy is to have good jobs and high wages, more than rapid innovation growth is required. But rather, *a substantial share of the subsequent manufacturing process entailed in supply of the innovative product must remain in the innovator country*. For example, the United States, though not the sole inventor of the automobile, was the creator of the inexpensive version of the vehicle that first attracted an abundant consumer market. Just as important, the United States was also the creator of the mass-production methods that made continuation of the automobile’s affordability possible.

Thus, the number of jobs provided by the invention process itself is usually minuscule, relative to those that are offered by the manufacturing sector when a particular innovation is produced for mass consumption. This remains true, despite the fact that there is
persistently growing labor productivity in the manufacturing sector. Today, the United States continues to be a leader in the invention process, but much of the manufacturing of its inventions is now outsourced to other countries, where reasonably skilled labor is available at prices far lower than in the United States.

A second and distinct observation follows from this. Americans continue to be large-scale consumers of manufactured goods—from automobiles to television sets and computers. If we do not make these products ourselves, we must trade for them. However since most trade—and, notably, most of America's imbalance in trade—is in manufactured goods, we are unlikely to be able to shift our negative balance toward a surplus solely by increasing our positive trade balance in services. Thus, unless they improve their country's manufacturing performance, Americans eventually will have to curtail their consumption of manufactured goods sharply, or else continue to go deeper into debt with America's trading partners. This process surely will not be allowed to go on indefinitely.

It follows from these considerations that continued good performance in the innovation process is not to be spurned. However, innovation by itself, without the retaining the resulting manufacturing, may not be enough to propel continued prosperity in the United States and other developed (i.e., dominant) nations.

**Toward remedies**

As the prior discussion makes clear, the United States must consider changes in its trade policy if it is to balance its imports and exports and ensure continued economic growth. As the discussion above has shown, in order to avoid being dominated a country must continue to produce a considerable portion of the world's traded goods. But what changes to the United States' trade position to produce such a balance are possible?

We will not recapitulate in any detail the well-known arguments against tariffs—that they may balance trade, but retaliatory tariffs are then likely to cause that rebalance to entail a lower economic level for both trading partners. Instead, we will focus on other possible courses of action that merit further investigation. For instance, the United States may wish to provide incentives for the domestic production of domestic inventions. As part of this, it may be advantageous to offer government support for early investment in innovative ventures and for training in the requisite technical and production skills required to manufacture both innovative traditional products in more productive ways.

There is also good reason to search for innovative new solutions to the ongoing US trade imbalance. For example, the United States should not reject out of hand Warren Buffett's proposal (Buffett 2003) to make use of the market mechanism for this purpose. An approach similar to carbon emissions entailing the use of tradable permits, which already has demonstrated considerable effectiveness, may prove helpful here. Such a scheme might award exporters saleable certificates that authorize the holder to import products with a market value that does not exceed that of the exports represented by the
certificates. The certificates could be traded among importers and exporters on a market, much like those used for the trading of carbon emissions allowances.

This approach already has proven itself reasonably effective and viable in the arena of environmental protection, and it seems reasonable to expect that it would perform effectively in balancing US trade. Surely it would ensure a reduction, if not the complete elimination, of trade imbalances and would provide automatic incentives for domestic manufacturers to avoid the added cost of purchasing import licensing certificates for components produced by outsourcing. Moreover, if such an import–export certificates market is sufficiently competitive, the added result will be a set of imports and exports that most effectively fits in with the demands of the consuming public in the United States.

In addition, when compared with tariffs, this approach has the peculiar distinction of succeeding in its mission of balancing trade, even if our trading partners adopt retaliatory import certificate programs. Should any trading partners choose to retaliate by adopting ordinary tariffs to restrict imports from the United States, that also would help, rather than hinder, this program by automatically lowering that country’s exports to the United States and, thereby, rebalancing trade. Thus, the incentive structure and likely consequences of such an import–export certificate scheme are radically different from those of a program of tariffs.

**Concluding comment**

Obviously there are other ways in which a country can protect itself from the damaging effects of a formerly dominated trading partner’s progress toward a dominant position. For example, as we already have argued elsewhere (Gomory and Baumol 2011), a dominated country’s entrance into industries in which it was formerly not capable of operating need not be seriously damaging to its trade partner. Intuitively, this result rests upon the fact that when both countries are able to supply and sell an identical product in a competitive world market, their prices must be the same—leaving consumers in each country the choice between the two sources.

This hints at the range of possibilities and the avenues open to a formerly dominant economy to preserve its prosperity. The central point of this chapter is to reiterate the importance of the issue discussed in this paper for the well-being of already dominant economies akin to that of the United States. It also is urgently important that we study the means by which other countries, such as China, can emerge from poverty without doing so at the expense of already prosperous economies and their people.

Clearly the most desirable outcome is a world in which both countries have a high standard of living. That can be attained, but it requires that the already developed country continue to be effective in enough industries to prevent its standard of living from being eroded by the emergence of the newly developed trading partner. This is represented by the middle of the hilltop, shown in Figure 2.1. It is not as advantageous for
the developed country as a trade partner that can be exploited, but it makes for a more stable world in which both trading partners spur each other on and develop more rapidly because the total resources and capabilities of the world are enhanced beyond those that were available in the previous, unbalanced situation.

Notes

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3. This common-sense observation is confirmed by direct calculation using the equations in appendix 1.
4. Here it is important to clarify a few matters regarding utility versus national income. If we consider a single equilibrium, it is natural to compare various bundles of goods by comparing their priced value. The result of the comparison is not affected by the choice of the monetary unit, as the value of two bundles of goods would be affected equally by change. However, in this article we are constantly comparing what a country obtains at different equilibria and, therefore, we encounter the well-known problem of the “numeraire.”

We have chosen to compare the bundles of goods a country obtains at the various equilibria by their utility, specifically Cobb–Douglas utility—the utility whose maximization produces the (Cobb–Douglas) demand that we use in our model. If we had chosen to use the priced value of the goods a country produces as a measure, we could take, for example, the goods the country consumes in autarky with fully developed industries and then compare the cost of that bundle at current equilibrium prices with the current national income. As with the consumer price index, different choices of a comparison bundle generally will produce different results. We have done calculations with this approach and have obtained diagrams that exhibit exactly the same features as Figure 2.1.

This is what we would expect, since we can show that for any chosen \( Z_1 \) (share of national income), there are a finite number of dominating equilibria—all clustered near the upper regional utility boundary. For any equilibrium having that \( Z_1 \), one of the cluster will provide the same or a greater amount of every good. So whatever measure is chosen, the maximizing equilibrium for \( Z_1 \) will be one of that cluster, and the maximizing boundary with that choice of measure will be close to that obtained by utility.

5. A fuller treatment of this is available in Gomory and Baumol (2000), and an even more rigorous treatment can be found in Gomory and Baumol (1998).
6. In Figure 2.1 the light, jagged line for Country 1 is the exact boundary in an eleven-industry model. The data for that model are listed in appendix 2. The other curves in Figure 2.1 are smoothed upper bounds.
7. Here we should bear in mind that, in general, the more productive equilibria are higher up in Figure 2.1, while the less productive ones are located lower down in the diagram. For any equilibrium, \( p \), for industries in which both countries can increase their productivities, there will be equilibria directly above \( p \) in the diagram that deliver as much or more of every good as \( p \) does. So, the Pareto optimal equilibria have the property that in every industry one or the other country is at its maximal productivity. These Pareto optimal equilibria can be shown to form a band that lies below and has the shape of the upper boundary. We have
discussed this as the “region of maximal productivity” in our earlier work (Gomory and Baumol 1998; Gomory and Baumol 2000: 102–103).

8. It may be noted, as a fact suggesting how much matters can change, that during these earlier years the standard joke was that products of questionable quality always bore the label “Made in Japan.”

9. This is the usual problem of the “numeraire.”

References


Appendix 1: Equilibrium equations

Notation

In our standard Ricardian model there are two countries and n industries. The quantity \( q_{i,j} \) of good \( i \) produced in Country \( j \) is determined by linear production function \( e_{i,j} l_{i,j} \) with \( l_{i,j} \) denoting the amount of labor employed in Country \( j \) in producing good \( i \). The size of labor force for each country is \( L_j \). Country \( j \)'s consumption of good \( i \) is denoted by \( y_{i,j} \) and its production of good \( i \) by \( q_{i,j} \). Country \( j \)'s production share or market share of world output of good \( i \) is represented by \( x_{i,j} = q_{i,j} / (q_{i,1} + q_{i,2}) \), so that the vector \( x = (x_{i,j}) \) describes the pattern of production.

Each of the two countries participating in trade has a given utility function of Cobb–Douglas form with demand parameters \( d_{i,j} \).

The price of good \( i \), \( p_i \), and \( w_j \), the wage in Country \( j \), and \( Y_j \) the national income of Country \( j \) are all expressed in monetary units. The standard equilibrium equations given below are homogenous in these variables. This means that if we choose a different monetary unit thus replacing any set of \( p_i \), \( w_j \), and \( Y_j \) values at an equilibrium by \( kp_i \), \( kw_j \), and \( kY_j \), these values too would satisfy the equilibrium conditions while leaving all the non-pecuniary variables such as \( q_{i,j} \) and \( l_{i,j} \) unchanged. We eliminate this ambiguity by choosing at each equilibrium the monetary unit, the MU, that makes the total world income \( Y_1 + Y_2 = 1 \) MU. The national incomes with this normalization we will refer to as \( Z_1 \) and \( Z_2 \). We will then have at each equilibrium uniquely determined pecuniary values and the uniquely determined national income, expressed in MU units, satisfy \( Z_1 + Z_2 = 1 \).

Equilibrium conditions

For any given vector of productivity parameters \( \varepsilon = \{e_{i,j}\} \) there is a stable equilibrium giving a national income \( Z_j \) and a utility \( U_j \) for each country.

The first equilibrium condition states that national income or consumption \( Z_j \) in Country \( j \) must equal the total value of the goods produced in Country \( j \). With a Cobb-Douglas utility each country spends \( d_{i,j} Z_j \) on good \( i \), so total world expenditure on the \( i \)th good is \( (d_{i,1} Z_1 + d_{i,2} Z_2) \). Since the fraction produced in each country is \( x_{i,j} \) the balance of the value of production and consumption requires for each country:

\[
\sum_i x_{i,j} (d_{i,1} Z_1 + d_{i,2} Z_2) = Z_j. \tag{1.1}
\]
Second, we have a zero-profit condition. World expenditure on Country j’s output of good i all goes into the wages of the labor \( l_{ij} \) employed in that industry, so:

\[ w_l l_{ij} = x_{ij} (d_{ij} Z_1 + d_{ij} Z_2) \]  

(1.2)

Third, is the full-employment requirement for each country. This is expressed as the condition that the wage rate times the country’s total labor force equals national income (the wage rate condition):

\[ w_i L_i - Z_i, \quad w_2 L_2 = Z_2 \]  

(1.3)

Fourth, we have the requirement that, for each good the value of the output of good i at the equilibrium price \( p_i \) equals the total amount consumers are willing to spend on it

\[ p_i (q_{i,1} + q_{i,2}) = d_{ij} Z_1 + d_{ij} Z_2 \quad \text{or} \quad \frac{p_i q_{i,1}}{w_i l_{ij}} = \frac{p_i q_{i,2}}{w_i l_{ij}} \]  

(1.4)

where the second form of (Equation 1.4) follows directly from the first by multiplying through by \( x_{ij} = q_{i,j}/(q_{i,1} + q_{i,2}) \) and using (Equation 1.2).

Finally, we have the conditions that require that in each industry production, or equivalently market share, is always assigned to the producer or producers with the lowest unit cost. For example, if in industry i \( w_i / e_{i,1} < w_2 / e_{i,2} \), then \( x_{i,1} = 1 \) and \( x_{i,2} = 0 \). More generally:

\[
\begin{align*}
\text{if Country 1 unit cost } (w_i / e_{i,1}) &< (w_2 / e_{i,2}) \text{ then } x_{i,1} = 1 \text{ and } x_{i,2} = 0 \\
\text{if Country 1 unit cost } (w_i / e_{i,1}) &> (w_2 / e_{i,2}) \text{ then } x_{i,1} = 0 \text{ and } x_{i,2} = 1 \\
\text{if unit costs are equal } (w_i / e_{i,1}) &= (w_2 / e_{i,2}) \text{ any } x_{i,1} + x_{i,2} = 1 \text{ is allowed.}
\end{align*}
\]

(1.5)

It is, of course, the actual producer’s unit cost that determines the price \( p_i \). The conditions (Equation 1.5) include the familiar comparative-advantage criterion.

Note that because of (Equation 1.1) the equilibrium conditions include balanced trade.

**APPENDIX 2: DATA FOR FIGURE 2.1**

Both countries’ labor forces are of size 1. \( L[1] = L[2] = 1 \).

Productivities of the two countries are given below.

\[ e[1] = \{0.97, 0.92, 0.72, 0.94, 0.84, 0.52, 0.73, 0.77, 0.53, 0.91, 0.89\} \]

\[ e[2] = \{0.52, 0.61, 0.91, 0.92, 0.84, 0.97, 0.83, 0.92, 0.31, 0.83, 0.72\} \]
Demands of the two countries (approximated to three decimal places) are given below.
\[ d[1] = \{0.033, 0.073, 0.0153, 0.047, 0.067, 0.105, 0.087, 0.113\} \]
\[ d[2] = \{0.081, 0.169, 0.113, 0.177, 0.032, 0.048, 0.097, 0.040\} \]

Appendix 3: Explaining the characteristic regional shape

In this appendix we offer a brief summary of the reasons why the calculations of equilibria always result in regions of equilibria with the characteristic shape shown in Figure 2.1. While the full reasoning takes up a good deal of space in Gomory and Baumol (1998), we will try to give the essentials behind that reasoning here.

Calculating the boundary: a linear programming problem

The height of the boundary curve for Country 1 at a given \( Z_1 \) value represents the upper bound of the utilities that Country 1 can obtain at any of the equilibria having that \( Z_1 \). In other words, if we can find a calculation that maximizes utility, subject to the constraint that we deal only with equilibria whose value is \( Z_1 \), and we then repeat that calculation for each \( Z_1 \), we will get the utility value represented by the Country 1 boundary curve.

Two helpful facts simplify computing these boundaries.

First: We do not need to deal in this calculation with all the equilibrium properties listed in appendix 1, but only with Equation 1.1, which asserts that the value of a country’s production must match the value of its consumption.

Second: We do not need to deal directly with Cobb–Douglas utility in doing the maximization. Rather we use a simplified version of it, called “linearized utility,” introduced in Gomory (1994), which gives the same value as Cobb–Douglas utility at all equilibria but is linear in the production variables \( x_{i,j} \).

These two facts turn the problem of maximizing utility over all equilibria with a given \( Z_1 \) into a much simpler problem: the problem of maximizing an objective function that is linear in the production variables \( x_{i,j} \), subject only to the constraint (Equation 1.1) for the given \( Z_1 \). However, the constraint (1.1) is also linear in the production variables \( x_{i,j} \). The problem of finding the boundary, therefore, can be recognized as an ordinary linear programming problem.

In fact, it is the simplest form of linear programming problem, the so-called “knapsack” problem. In the usual knapsack problem we have an assortment of object having given values and given weights. The problem is to find the choice of objects that can be put into the knapsack without exceeding the weight limit. In our boundary problem the objects we
put into the knapsack are industries whose production is assigned to Country 1, and the weight limit is the specified national income, $Z_1$. Country 1 cannot have an assignment of industries that exceeds its $Z_1$, and the corresponding constraint holds for Country 2.

### The Classical Level

If both countries are fully developed (i.e., $e_{ij} = e_{ij}^{\text{max}}$), the equilibrium that results is called “the classical equilibrium.” We refer to the $Z_1$ value of the classical equilibrium as the classical level, $Z^c$.

We show in Gomory and Baumol (1998) what is intuitively plausible in any case—that Country 1 cannot get a larger share of world output (i.e., $Z_1 > Z^c$) by its own efforts. It is, after all, already fully developed at $Z^c$. To obtain an equilibrium with $Z_1 > Z^c$, Country 1 must acquire industries in which its trading partner is not fully developed; Country 1 would not be the cheaper producer in these industries if its trading partner were fully developed.

On the other hand, for $Z_1$ values less than $Z^c$ there are enough industries in which Country 1 is the cheaper producer so that if they were all assigned to Country 1, it would have a total production that would exceed $Z_1$. So the problem for Country 1 is to choose the subset of these industries that maximize utility.

### The Derivative of the Boundary Curve in a Special Case

If the countries are of equal size ($L_1 = L_2$) and have the same demand structure ($d_{i,1} = d_{i,2}$), the boundary equilibria (those with highest utility) will always be obtained by simply assigning industries to Country 1 in the order of Country 1’s (fully developed) comparative advantage, until the limit $Z_1$ is reached. This is a very simple way to solve the maximization problem and obtain the boundary curve.

In this case the derivative of the boundary $dU_1/dZ_1$ is given by

$$
\frac{1}{U_1} \frac{dU_1}{dZ_1} = \frac{1}{Z_i} \ln \frac{e_{i,1}^{\text{max}} Z_2}{e_{i,2}^{\text{max}} Z_1} = \frac{1}{Z_i} \ln \frac{e_{i,1}^{\text{max}} W_2}{e_{i,2}^{\text{max}} W_1}
$$

(3.1)

where the index $i$ designates the last industry that was assigned in whole or in part to Country 1 and caused it to reach the limit $Z_1$.

The term $1/Z_1$ on the right is always positive and reflects the fact that with increasing $Z_1$, Country 1’s share of the output of every industry is $d_{i,1} Z_1$, which grows with $Z_1$.

The next term reflects the impact of the industry that is currently being moved from Country 2 to Country 1. If $Z_1 < Z^c$ this will still be an industry in which Country 1 is
the cheaper producer, \((i.e., e_{i,1}^{\text{max}}/w_1 > e_{i,2}^{\text{max}}/w_2)\) and the term inside the logarithm in (Equation 3.1) will be greater than 1, so that this term will be positive.

Since both terms on the right are positive, the move underway will have a positive impact on the utility of Country 1. So, for all \(Z_1\) values that are < \(Z_C\), the derivative of utility will be positive and the boundary curve will be sloping upward.

However, for \(Z_1 > Z_C\), Country 1 is acquiring industries in which it would not be the cheaper producer if its trading partner were fully developed. Therefore the logarithm term will be negative and will start to counteract the positive effect of the term \(1/Z_1\). Eventually, as \(1/Z_1\) grows smaller with increasing \(Z_1\) and the second term becomes more negative with decreasing comparative advantage, the total on the right will turn negative and the boundary curve will have negative slope and will turn downward. This is, in fact, the behavior we see in Figure 2.1.

The derivative of the boundary curve in the general case

It is still possible to obtain the derivative of the boundary in the general case without solving the linear programming problem. This is explained in Gomory and Baumol (1998). The result is

\[
\frac{1}{U_1} \frac{dU_1}{dZ_1} = (1 - \gamma(x)) \left[ \frac{1}{Z_1} + \frac{d_{i,1}}{TD_i} \ln \left( \frac{e_{i,1}^{\text{L}} Z_1}{e_{i,2}^{\text{L}} L_i Z_i} \right) \right]
\]

(3.2)

Despite the apparent increased complexity the derivative in the general case has the same qualitative features as the special case, described above.

The first term in parentheses is always positive, as \(\gamma(x)\) is a measure of the asymmetry of the demands and cannot exceed 1. So the sign of the derivative is always determined by the bracket term.

Within the bracket, \(1/Z_1\) is always positive, and as \(TD_i = (d_{i,1} Z_1 + d_{i,2} Z_2)\) is the total worldwide demand for the \(i\)th product, it too is positive. The sign of the logarithm term again is determined by which country is the cheaper producer in the industry that is shifting. As before, the logarithm term is positive for \(Z_1 < Z_C\) and increasingly negative for \(Z_1 > Z_C\).

So, in general, the boundary curve for Country 1 ascends up to \(Z_C\) and continues upward until the gain from increasing world share for Country 1 is outweighed by the decreasing effect on world output, which is caused by Country 1 being the producer in many industries in which it is not the (potentially) best producer. At this point, the curve turns downward.

This is all a direct consequence of the equations of equilibrium.